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When is a cohort not a cohort?

Spurious parameters in stochastic longevity models

Jon Palin

8 September 2016

Overview

- Many models assume a structure for mortality in terms of age, period and cohort parameters
- But the parameters are not always what they seem ...
- … e.g. cohort parameters may not reflect cohort effects.
- When the parameters are misleading, the projections are unlikely to be correct.

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Agenda

- Why include cohort parameters?
- Fitting the APC model
- Synthetic data
- Spurious parameters
- Causes of spurious parameters
- Implications for projections
- Models with plausible parameters
- Summary

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WHY INCLUDE COHORT PARAMETERS?

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Cohort effect

- "The generations born between 1925 and 1945 […] have experienced more rapid improvement than earlier and later generations. This feature […] is sometimes referred to as the U.K. 'cohort effect'."
  - Willets (2004) "The Cohort Effect: Insights and Explanations"

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Mortality improvements for UK males

	1970-75	1975-80	1980-85	1985-90	1990-95	1995-00	2000-05	2005-10
85-89	-0.1%	1.0%	0.7%	1.8%	0.7%	1.6%	1.8%	2.6%
80-84	-0.4%	0.9%	0.8%	2.1%	1.1%	2.3%	2.8%	3.0%
75-79	0.2%	1.0%	1.3%	2.4%	1.8%	2.4%	3.4%	3.8%
70-74	0.7%	1.6%	1.4%	2.6%	1.5%	3.2%	4.1%	3.3%
65-69	1.4%	1.5%	1.6%	1.8%	2.6%	3.9%	3.5%	3.0%
60-64	1.4%	1.2%	1.2%	3.1%	3.1%	3.1%	2.8%	3.3%
55-59	1.3%	0.3%	2.8%	3.7%	2.7%	2.5%	3.1%	1.4%
50-54	0.2%	1.5%	3.4%	3.2%	2.4%	2.1%	1.2%	2.4%
45-49	0.7%	2.2%	2.5%	2.3%	2.5%	1.0%	1.5%	1.8%
40-44	1.7%	1.5%	2.5%	1.9%	0.0%	1.5%	0.9%	-0.3%

- Willets (2004) method applied to HMD data.
- Highest improvement in each period highlighted.

Cohort parameters in longevity models

- “Incorporation of cohort effects: This is important if we believe that cohort effects are present and need to be allowed for”
  - Cairns et al (2009)
- “the introduction of cohort terms generally leads to an improvement in overall fit, but can also make forecasting with these models problematic.”
  - Currie (2016)
- “The fact that parameters can be estimated does not imply that they can sensibly be interpreted”
  - Goldstein (1979)

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FITTING THE APC MODEL

Overview of the APC model

- Mortality is modelled by a combination of age, period and cohort parameters:
  - $\log m_{x,t} = \alpha_x + \kappa_t + \gamma_{t-x}$ (Currie, 2006)

- This is an age-period-cohort model for mortality **rates**.
- This is distinct from age-period-cohort models for mortality **improvements** (e.g. the CMI Model).

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Overview of the APC model

- Mortality is modelled by a combination of age, period and cohort parameters:
  - $\log m_{x,t} = \alpha_x + \kappa_t + \gamma_{t-x}$ (Currie, 2006)
- Fit the model to historical data to determine parameters.
- Choose a time series for κ_t (and maybe γ_{t-x}) and project.
  - Some concerns about how this is done – but not today's topic.

- See e.g. Cairns et al (2009) for technical details.

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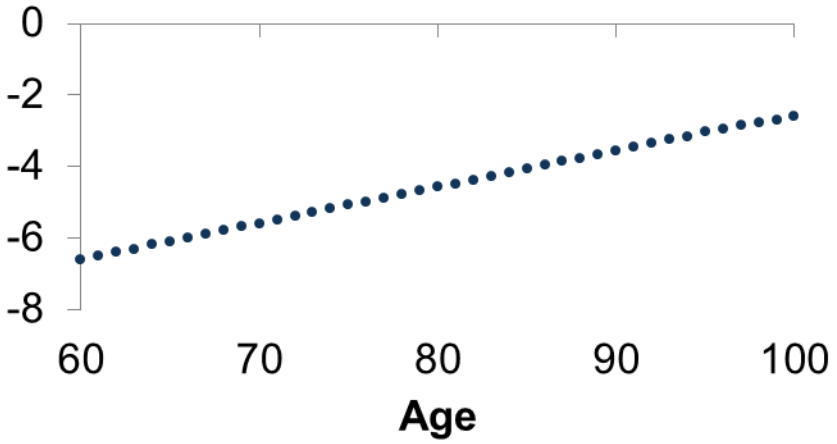
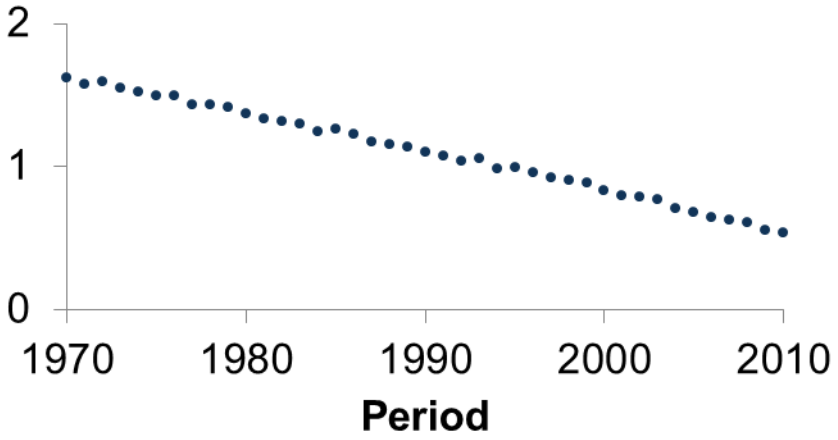
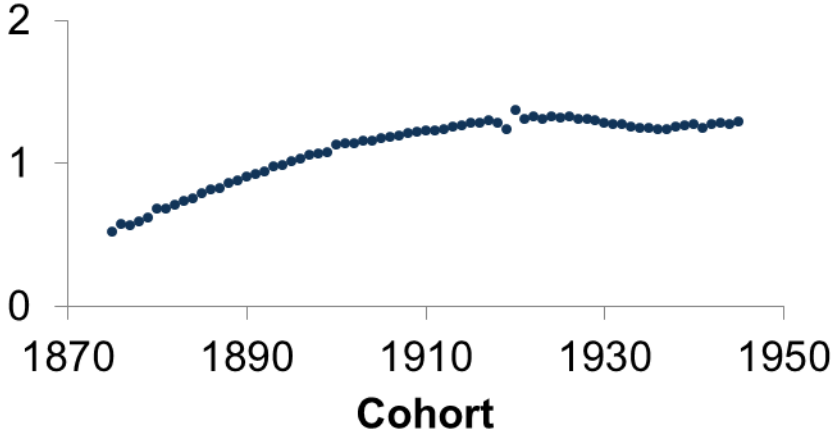
Sample parameters for the APC model

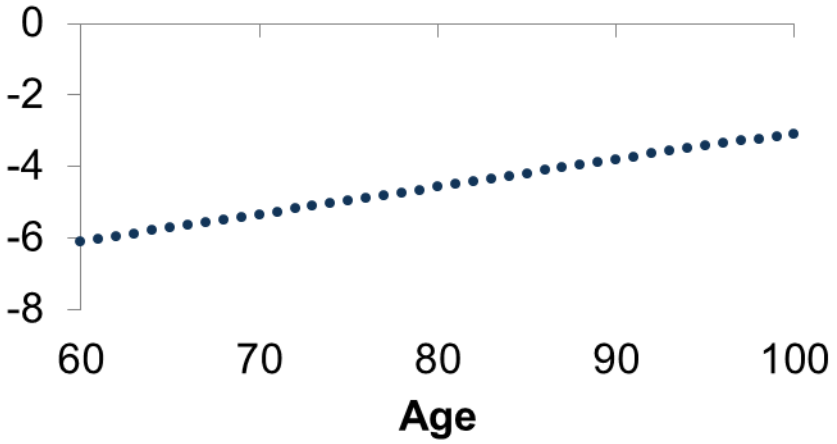


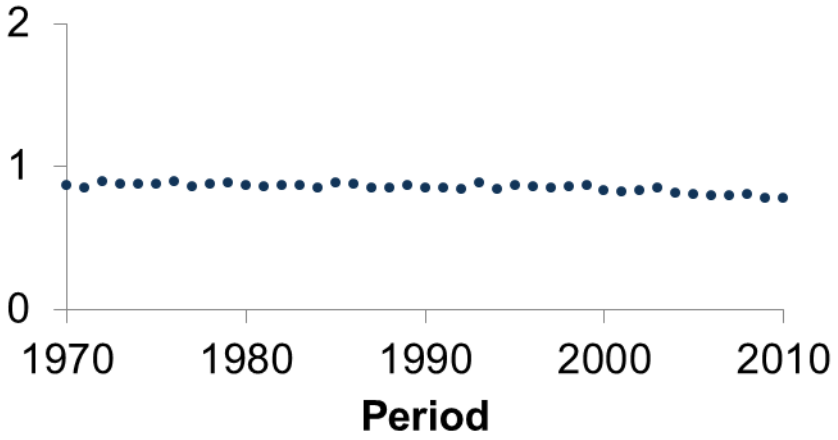


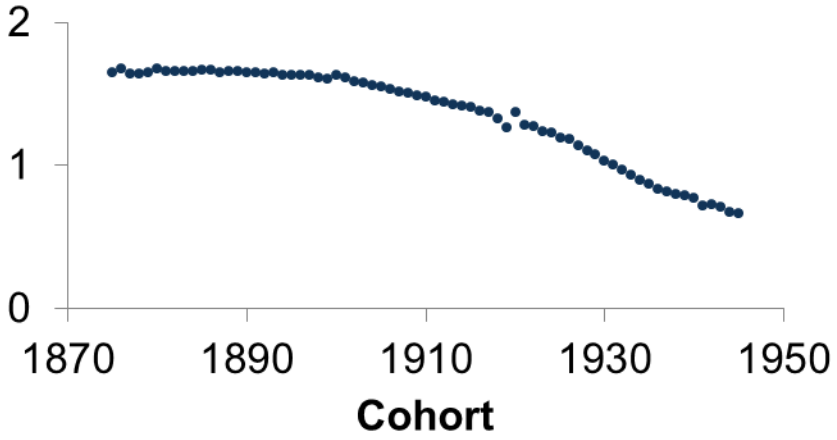


- Fitted to HMD data for UK males (GBR_NP); ages 60-100; years 1970-2010
- Ignoring cohorts with five or fewer observations

Alternative parameters for the APC model







- Different parameter values
- Same mortality rates
- Same goodness-of-fit

Identifiability

- Many models, including APC, are not “identifiable” – there are multiple sets of parameters that give the same fit.
- e.g. the following transformations do not affect $\log m_{x,t}$

$$\begin{array}{rcrrrr} \alpha_x & \rightarrow & \alpha_x & +\theta_1 & +\theta_2 & +\theta_3 x \\ \kappa_t & \rightarrow & \kappa_t & -\theta_1 & & -\theta_3 t \\ \gamma_{t-x} & \rightarrow & \gamma_{t-x} & & -\theta_2 & +\theta_3 (t-x) \end{array}$$

- It is customary to impose “identifiability constraints” e.g.

$$\sum \kappa_t = \sum \gamma_{t-x} = \sum \gamma_{t-x}(t-x) = 0$$

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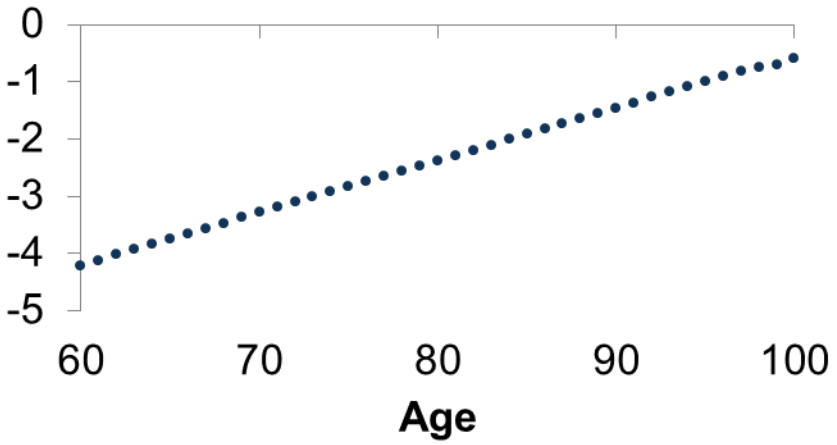
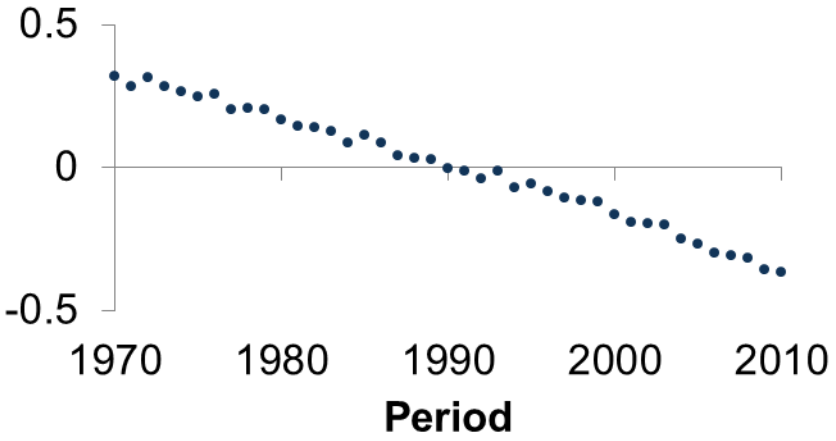
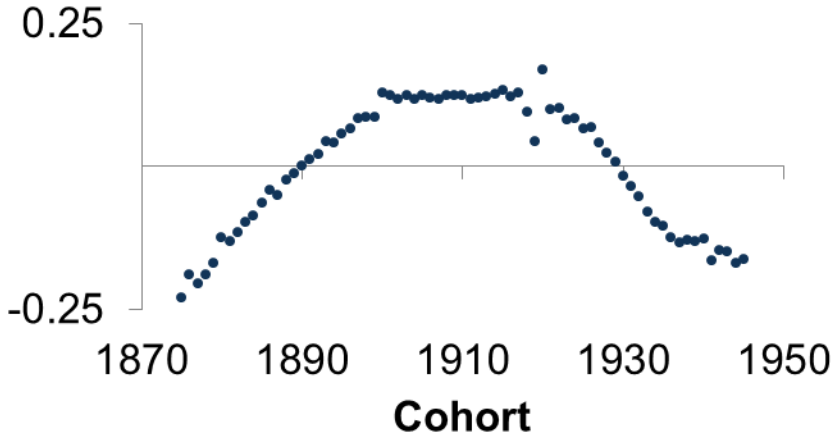
After imposing identifiability constraints







- Period and cohort terms are zero on average.
- No linear slope to the cohort terms.

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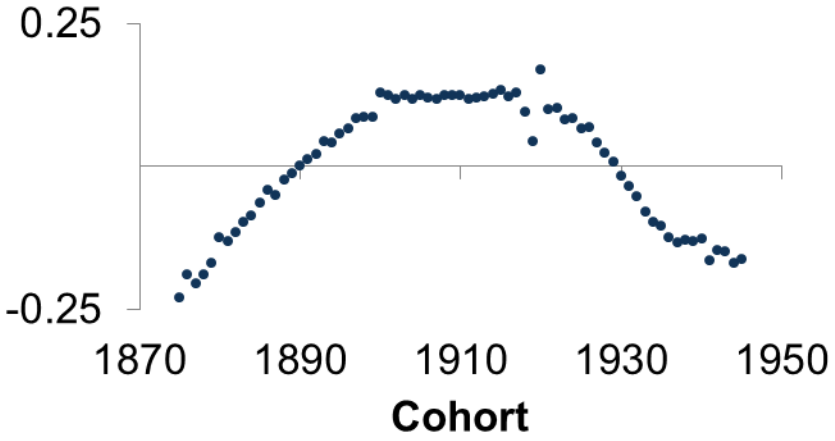
Are the parameters genuine?

i.e. do **cohort parameters** reflect **cohort effects**?



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SYNTHETIC DATA

Synthetic data

- Real data is messy; so look at a simpler case first.
- Synthetic data allows for algebra, identities, proofs.

- Construct synthetic data without any cohort effects, and see how the APC model fits that data.

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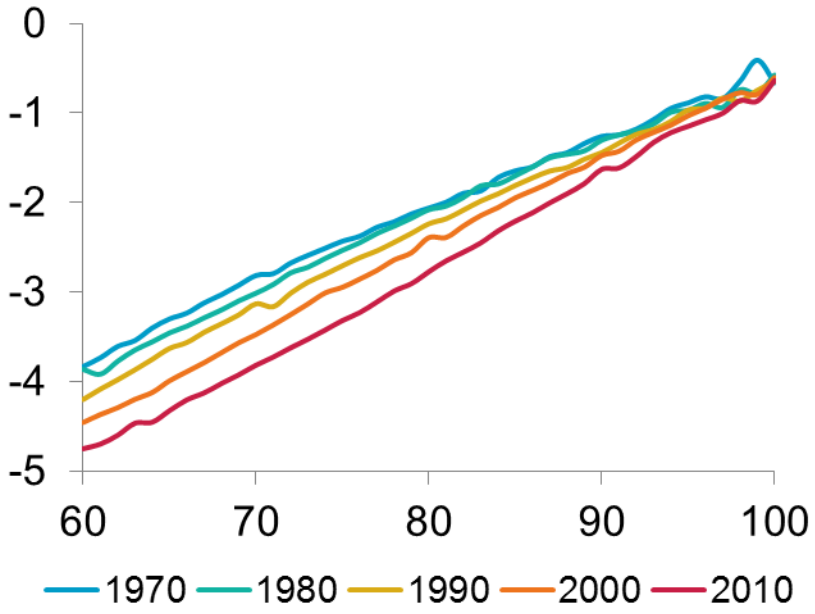
Constructing synthetic data

Historical mortality rates

Crude $\log m_{x,t}$ for UK males



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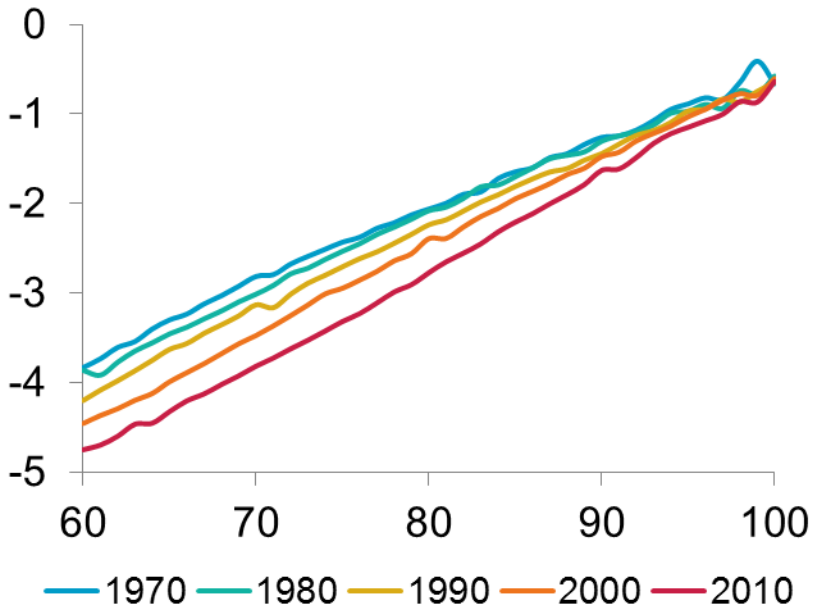
Constructing synthetic data

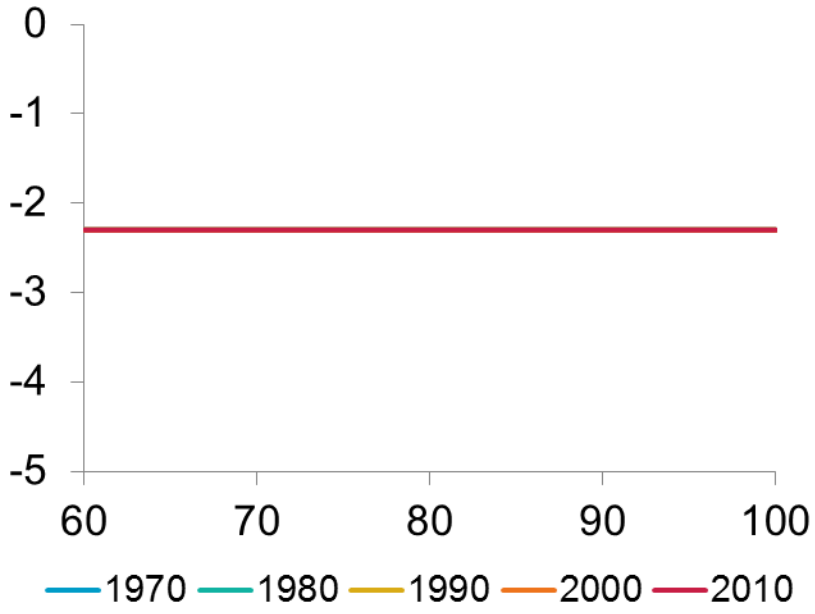
Historical mortality rates

Crude $\log m_{x,t}$ for UK males



Synthetic rates

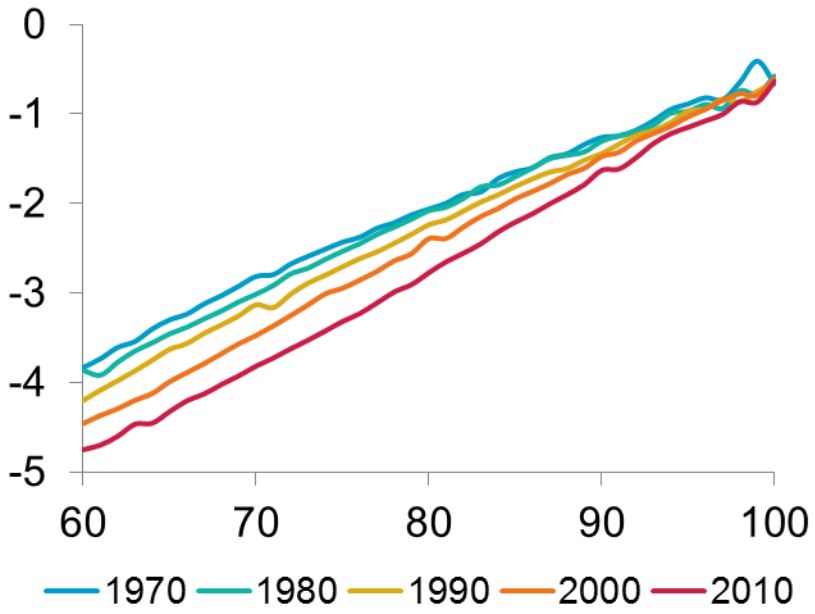
$\log m_{x,t} = A$



Constructing synthetic data

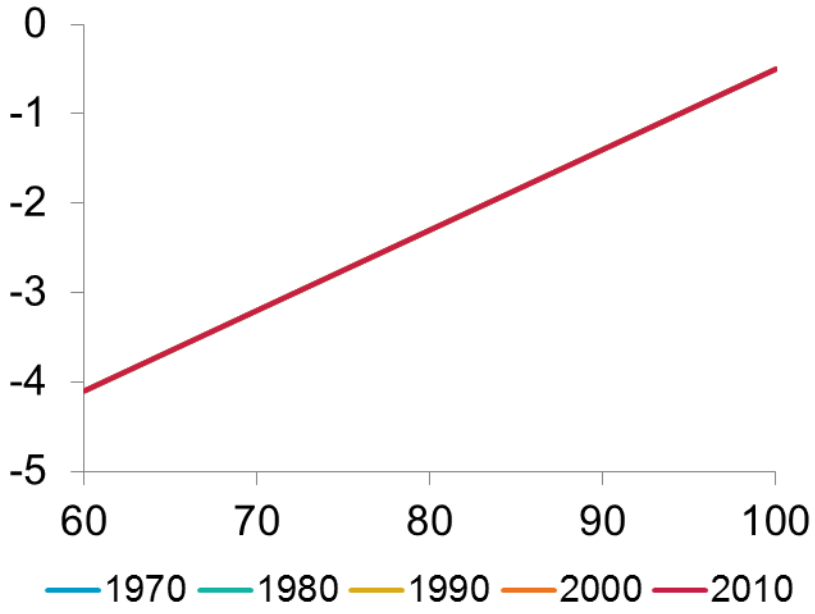
Historical mortality rates

Crude $\log m_{x,t}$ for UK males



Synthetic rates

$$\log m_{x,t} = A + B(x - \bar{x})$$



Constructing synthetic data

Historical mortality rates

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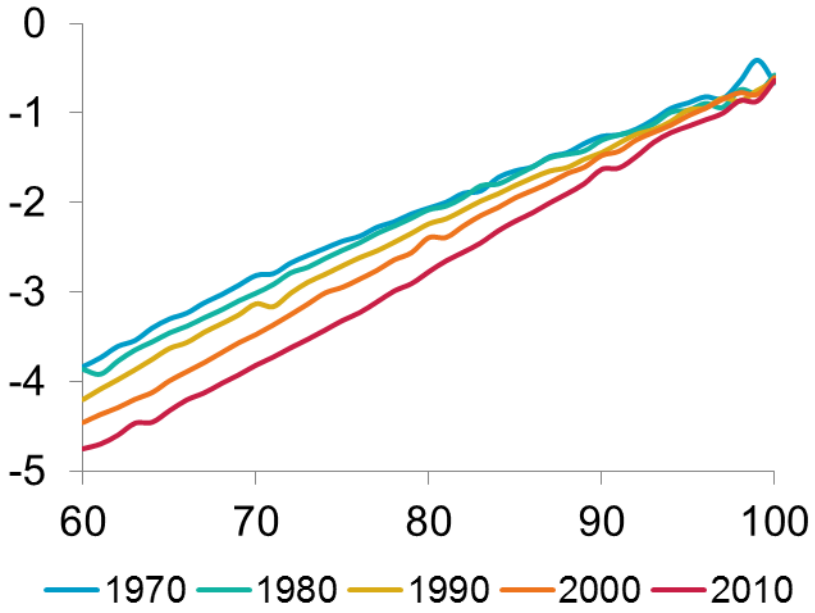
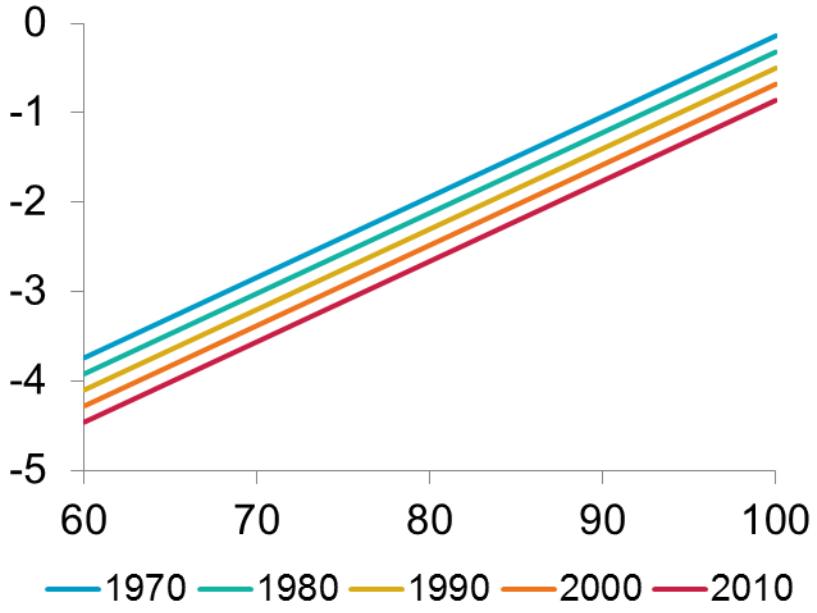


Synthetic rates

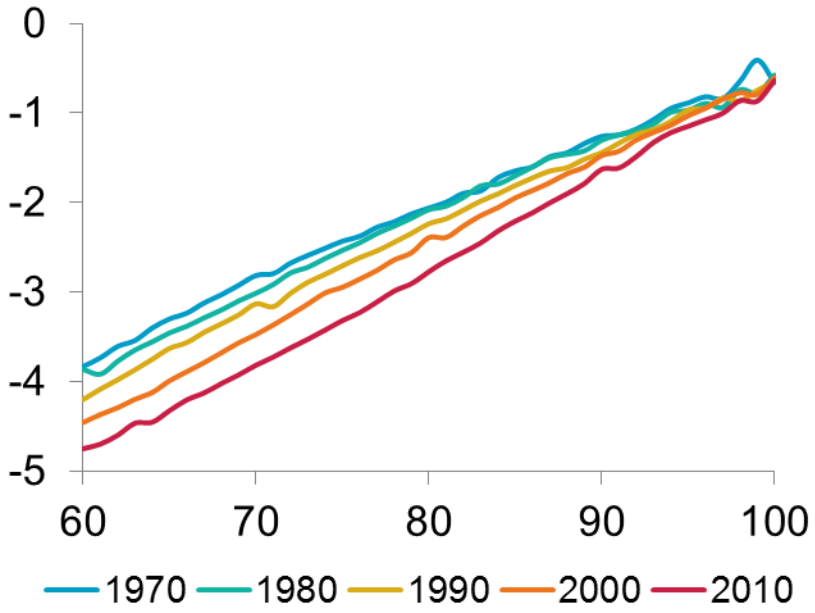
$$\log m_{x,t} = A + B(x - \bar{x}) - C(t - \bar{t})$$



Constructing synthetic data

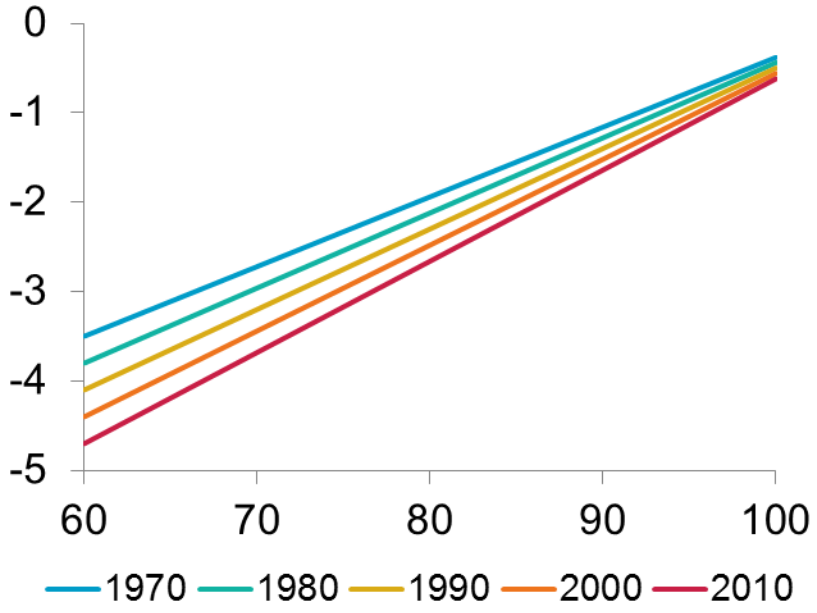
Historical mortality rates

Crude $\log m_{x,t}$ for UK males



Synthetic rates

$$\log m_{x,t} = A + B(x - \bar{x}) - C(t - \bar{t}) + D(x - \bar{x})(t - \bar{t})$$



Features of synthetic data

- Formula for synthetic data:
  - $\log m_{x,t} = A + B(x - \bar{x}) - C(t - \bar{t}) + D(x - \bar{x})(t - \bar{t})$
- Mortality improvement
  - Defined as $MI_{x,t} := \log m_{x,t-1} - \log m_{x,t}$
  - In our case $MI_{x,t} = C - D(x - \bar{x})$
  - So it only depends on age.
  - There is no cohort effect in the synthetic data.

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Constructing synthetic data

Parameter values

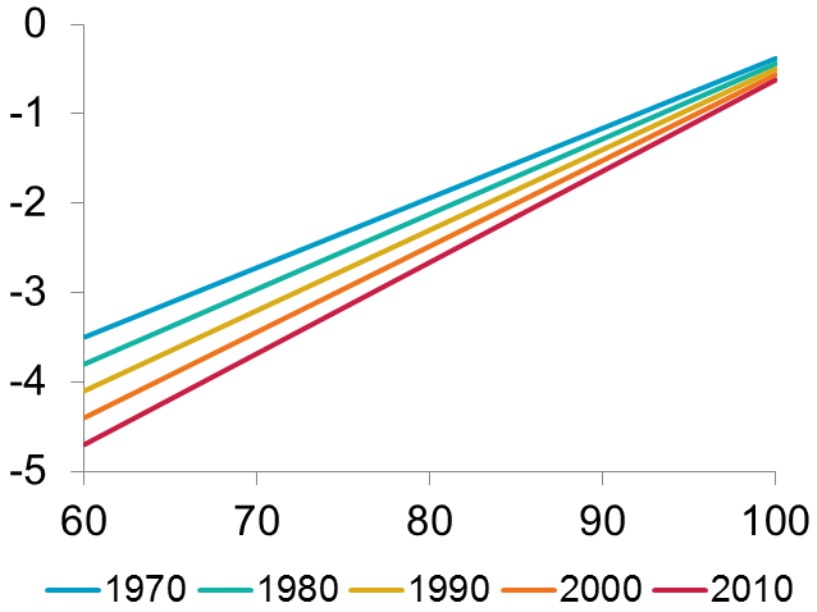
(to mimic UK males data)

- $A = -2.3$ (so $m_{80,1990} \approx 10\%$)
- $B = 0.09$
- $C = 1.8\%$
- $D = 0.06\%$

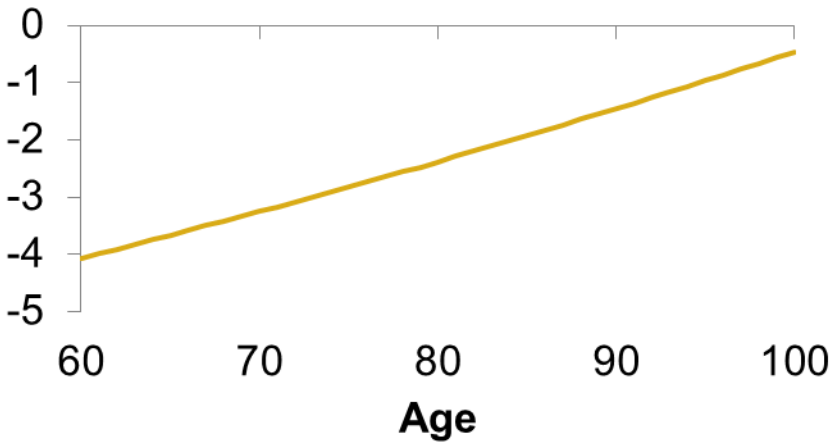
Age x	60	80	100
$MI_{x,t}$	3.0%	1.8%	0.6%

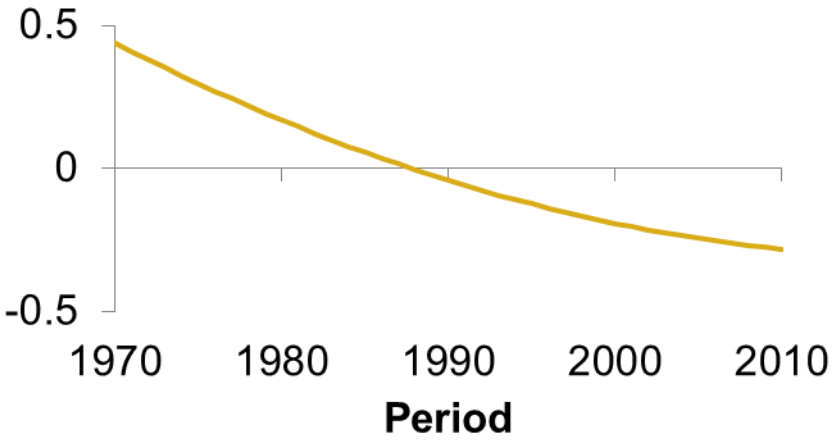
Synthetic rates

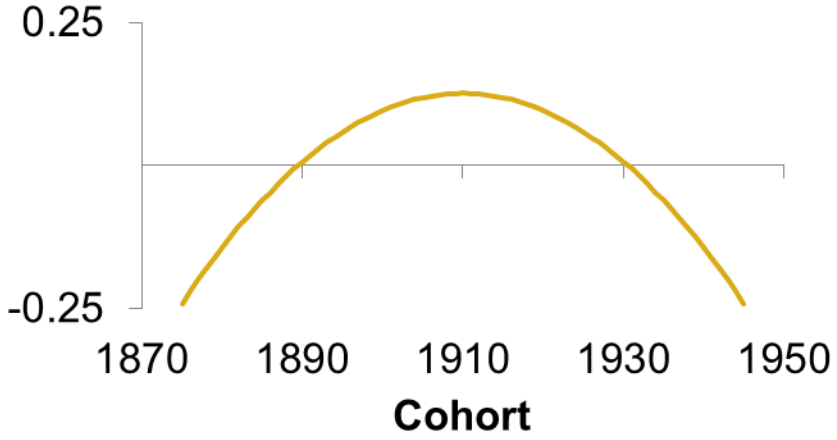
$$\log m_{x,t} = A + B(x - \bar{x}) - C(t - \bar{t}) + D(x - \bar{x})(t - \bar{t})$$



APC model: fitted to synthetic data

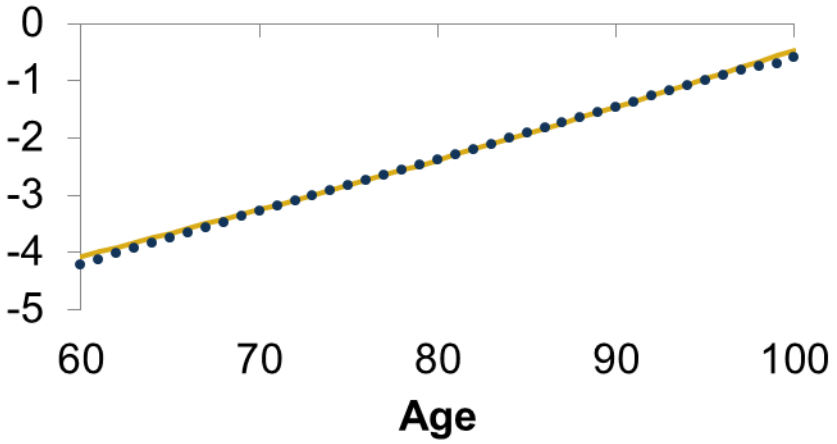


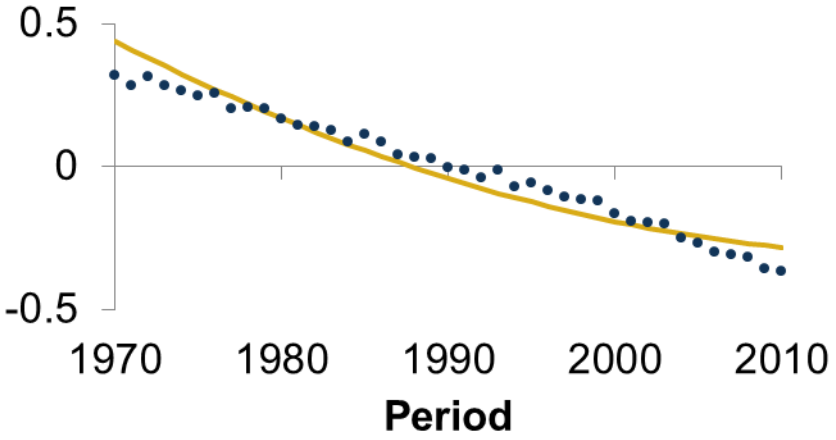


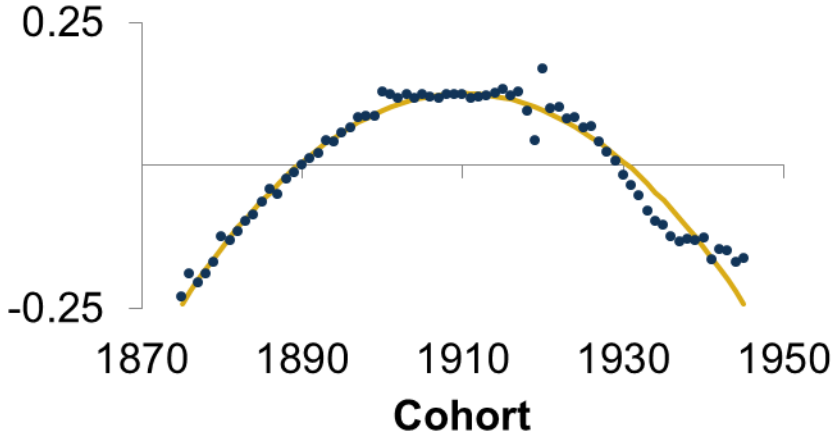


- We see non-zero cohort parameters even though there are no cohort effects in the synthetic data
- “Spurious” parameters

APC model: synthetic data and UK males







- Cohort parameters for the real data are very similar to those for the synthetic data
- So they are spurious too

SPURIOUS PARAMETERS IN VARIOUS MODELS

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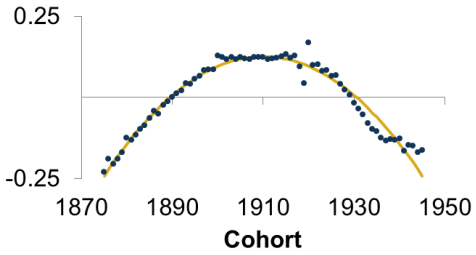
Models



- The next four slides show cohort parameters for various models, fitted to various sets of data, in published papers:
  - APC model
  - Lee-Carter with cohort
  - Cairns-Blake-Dowd M6
  - Plat
- For ease of comparison, cohort parameters for the APC model, fitted to synthetic data and UK males, are shown at top-right.

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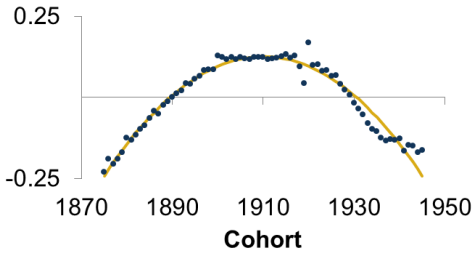
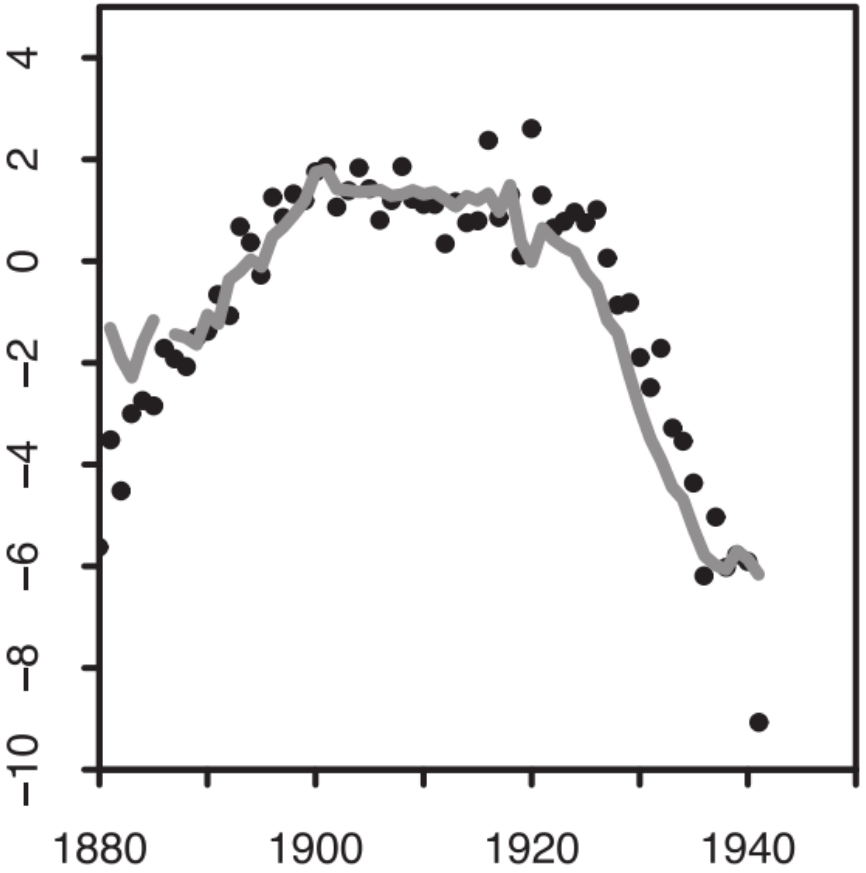
APC



- Figure 1 of Cairns et al (2011) – two different data sets.



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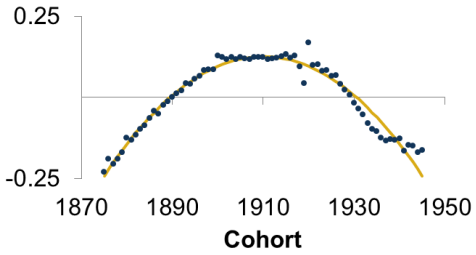
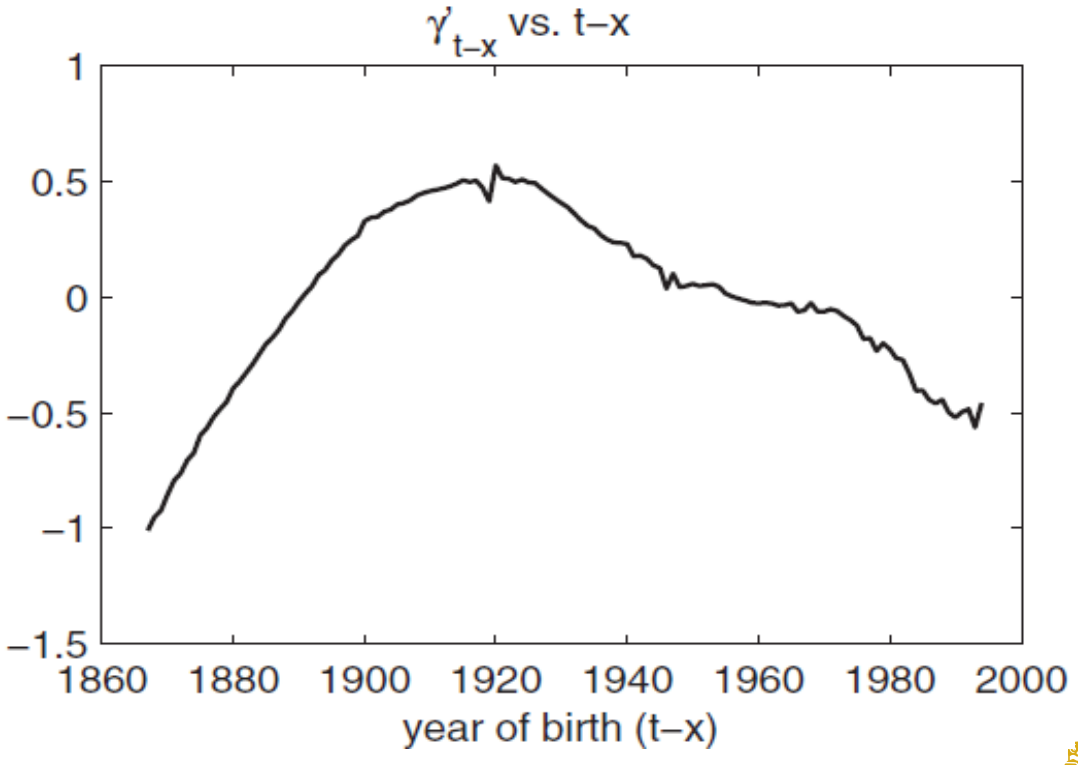
Lee-Carter with cohort



- Figure 7 of Villegas and Haberman (2014)



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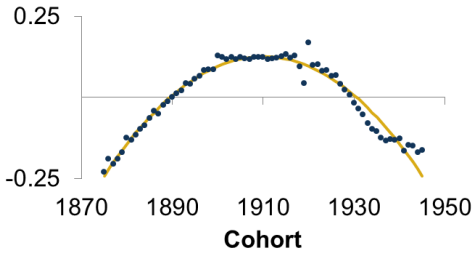
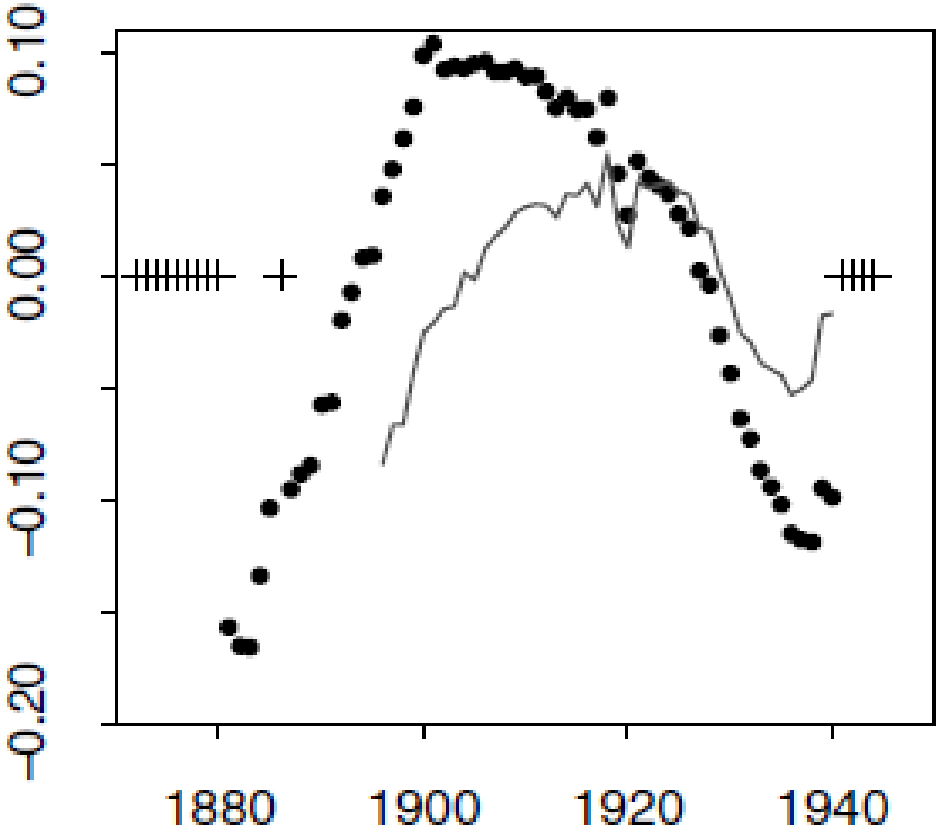
Cairns Blake Dowd M6



- Figure 7 of Cairns et al (2009) – two different time periods



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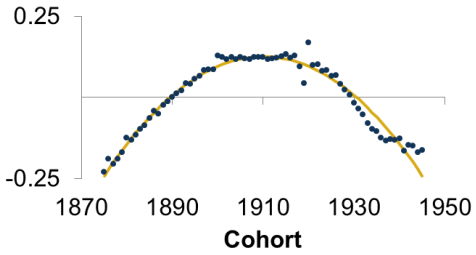
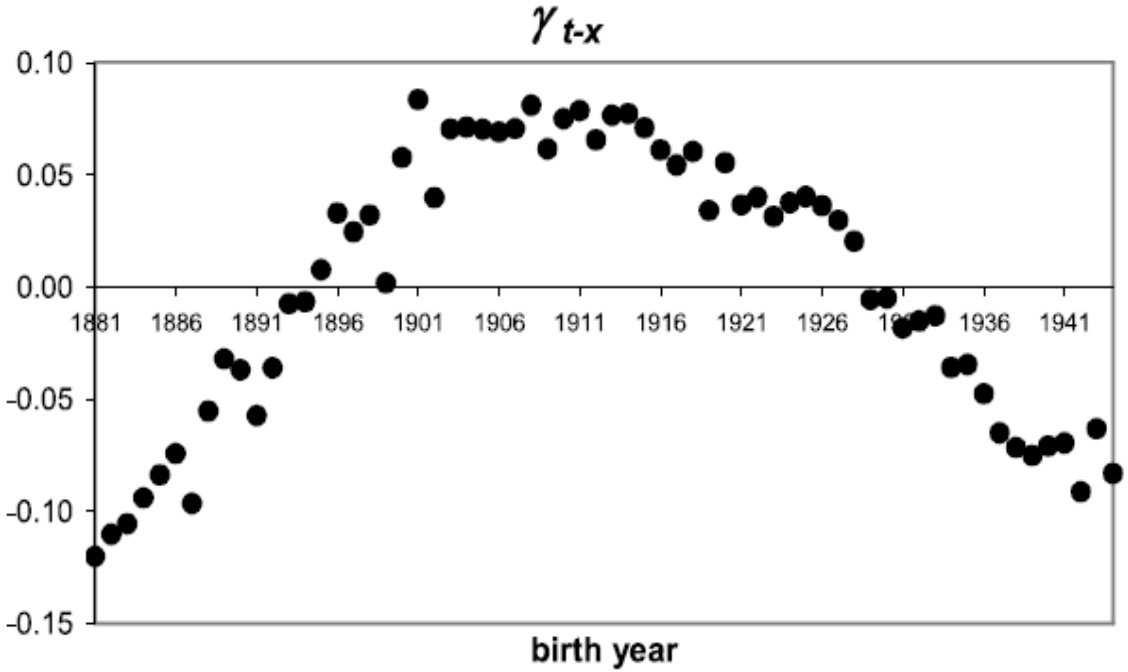
Plat



- Figure 2 of Plat (2009)



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Cohort parameters in various models

- Use of synthetic data suggests that the APC cohort parameters are spurious, and have a distinctive concave quadratic shape.
- Cohort parameters for these models have a similar shape:
  - Lee-Carter with cohort
  - Cairns-Blake-Dowd M6
  - Plat
- So these parameters are likely to be largely spurious too.

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CAUSES OF SPURIOUS PARAMETERS

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Why do we see spurious parameters?

- Consider this in terms of
  1. Mortality improvements
  2. Mortality rates
- Try to fit synthetic data with

$$\log m_{x,t} = A + B(x - \bar{x}) - C(t - \bar{t}) + D(x - \bar{x})(t - \bar{t})$$

$$MI_{x,t} = C - D(x - \bar{x})$$

- into the APC model structure with

$$\log m_{x,t} = \alpha_x + \kappa_t + \gamma_{t-x}$$

$$MI_{x,t} = \kappa_{t-1} - \kappa_t + \gamma_{t-x-1} - \gamma_{t-x}$$

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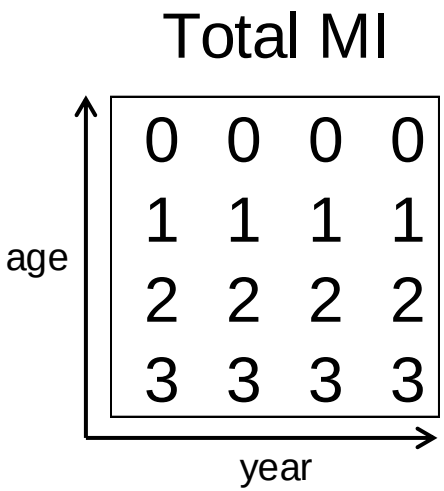
1. Consider mortality improvements

- Improvements in the synthetic data vary only by age.



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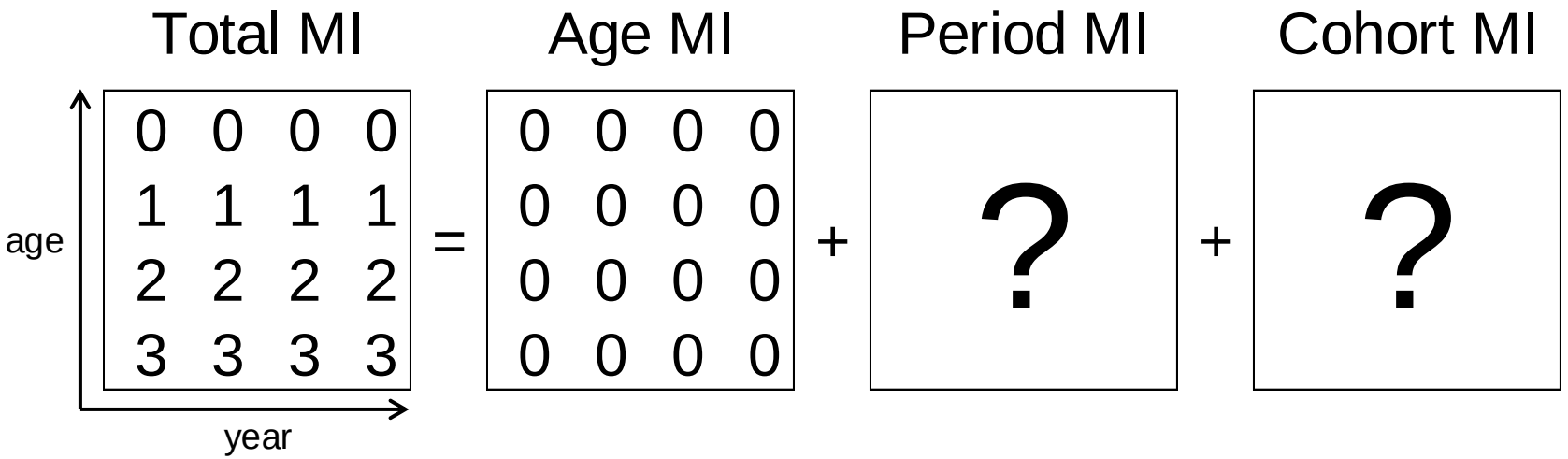
1. Consider mortality improvements

- Improvements in the synthetic data vary only by age.
- But the APC model doesn't have an age component of improvements.

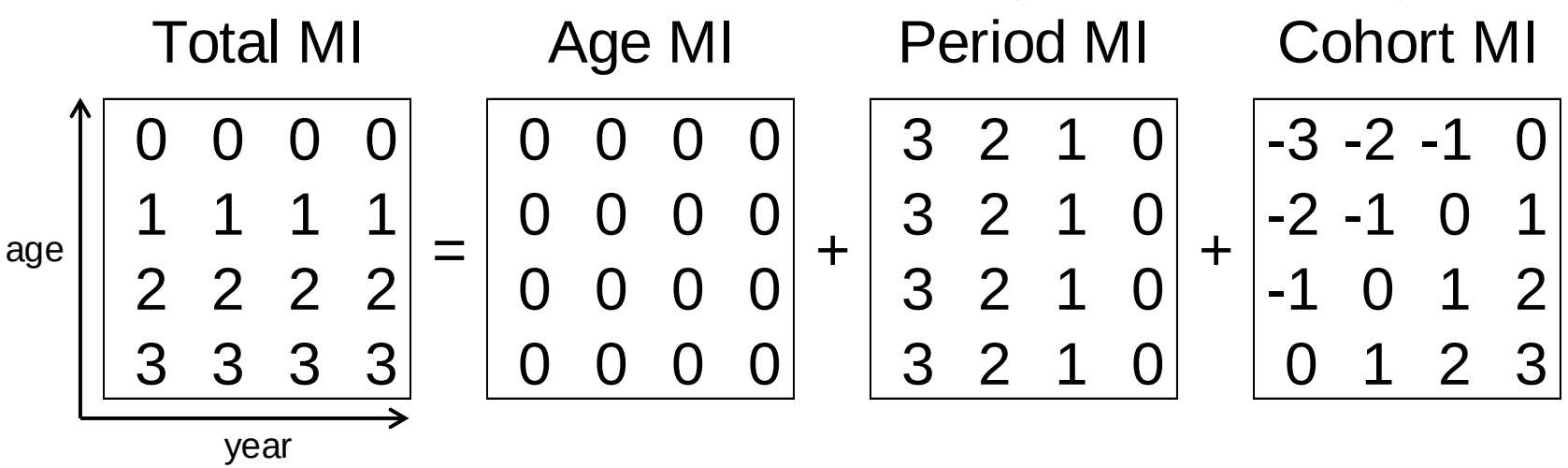


1. Consider mortality improvements

- Improvements in the synthetic data vary only by age.
- But the APC model doesn't have an age component of improvements.
- So "cheat" by using a combination of period and cohort.

1. Consider mortality improvements

- $C - D(x - \bar{x}) = [C - D(t - \bar{t})] + [D(t - \bar{t}) - D(x - \bar{x})]$



2. Consider mortality rates

- Modelling the synthetic data within the APC model:

$$\alpha_x =$$

$$\kappa_t =$$

$$\gamma_{t-x} =$$

2. Consider mortality rates

- Modelling the synthetic data within the APC model:

$$\alpha_x = \boldsymbol{A} + \boldsymbol{B}(\boldsymbol{x} - \bar{\boldsymbol{x}})$$

$$\kappa_t = -\boldsymbol{C}(\boldsymbol{t} - \bar{\boldsymbol{t}})$$

$$\gamma_{t-x} =$$

- How can we cope with $D(x - \bar{x})(t - \bar{t})$?
  - No multiplicative terms in the APC model

2. Consider mortality rates

- Key insight:

$$(t-x)^2 = t^2 - 2xt + x^2$$

- Re-arranging:

$$xt = \frac{1}{2}t^2 + \frac{1}{2}x^2 - \frac{1}{2}(t-x)^2$$

- We can create the multiplicative term "xt" by combining squares of age, period and cohort terms

2. Consider mortality rates

- Modelling the synthetic data within the APC model:

$$\alpha_x = \boldsymbol{A} + \boldsymbol{B}(\boldsymbol{x} - \bar{\boldsymbol{x}}) + \frac{1}{2}D(x - \bar{x})^2$$

$$\kappa_t = -\boldsymbol{C}(\boldsymbol{t} - \bar{\boldsymbol{t}}) + \frac{1}{2}D(t - \bar{t})^2$$

$$\gamma_{t-x} = \underbrace{-\frac{1}{2}D((t - x) - (\bar{t} - \bar{x}))^2}_{\text{Total} = \boldsymbol{D}(\boldsymbol{x} - \bar{\boldsymbol{x}})(\boldsymbol{t} - \bar{\boldsymbol{t}})}$$

2. Consider mortality rates

- Modelling the synthetic data within the APC model:

$$\alpha_x = \boldsymbol{A} + \boldsymbol{B}(\boldsymbol{x} - \bar{\boldsymbol{x}}) + \frac{1}{2}D(x - \bar{x})^2 + \frac{1}{24}D\left({N_t}^2 - {N_c}^2\right)$$

$$\kappa_t = -\boldsymbol{C}(\boldsymbol{t} - \bar{\boldsymbol{t}}) + \frac{1}{2}D(t - \bar{t})^2 - \frac{1}{24}D\left({N_t}^2 - 1\right)$$

$$\gamma_{t-x} = \underbrace{-\frac{1}{2}D((t - x) - (\bar{t} - \bar{x}))^2}_{\text{Total} = \boldsymbol{D}(\boldsymbol{x} - \bar{\boldsymbol{x}})(\boldsymbol{t} - \bar{\boldsymbol{t}})} \underbrace{+\frac{1}{24}D\left({N_c}^2 - 1\right)}_{\text{Total} = \boldsymbol{0} \text{ (for identifiability)}}$$

N_t and N_c are the numbers of periods and cohorts.

The shape of cohort parameters

- APC cohort parameters, when fitted to synthetic data, are driven by the slope of mortality improvements by age, D

$$\gamma_{t-x} = -\frac{1}{2}D((t-x)-(\bar{t}-\bar{x}))^2+\frac{1}{24}D\left({N_c}^2-1\right)$$

- If improvements are flat by age, then $D = 0$ and $\gamma_{t-x} = 0$
- But for countries which have had a roughly linear pattern of improvements by age, we see (roughly) a parabola for cohort terms.

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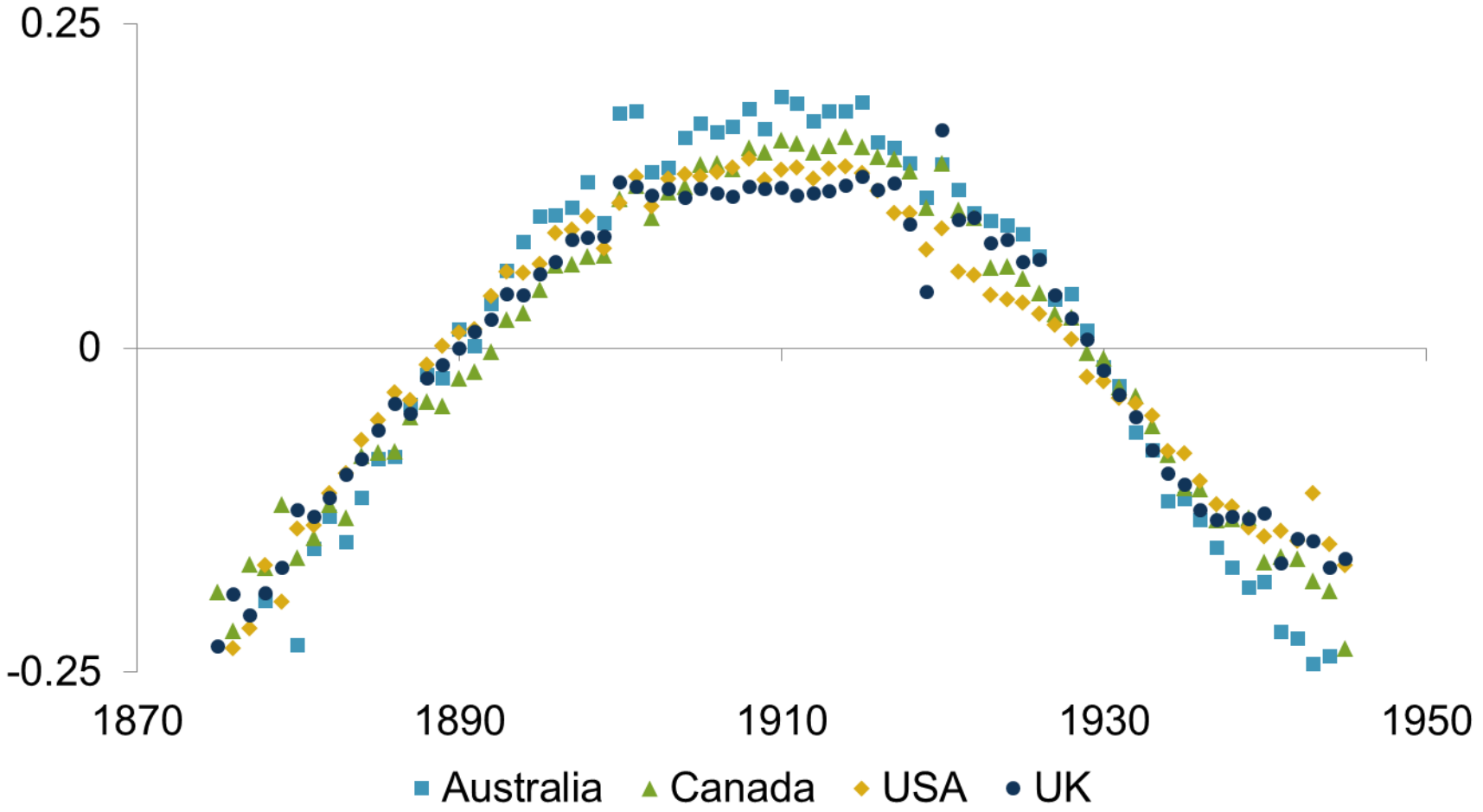
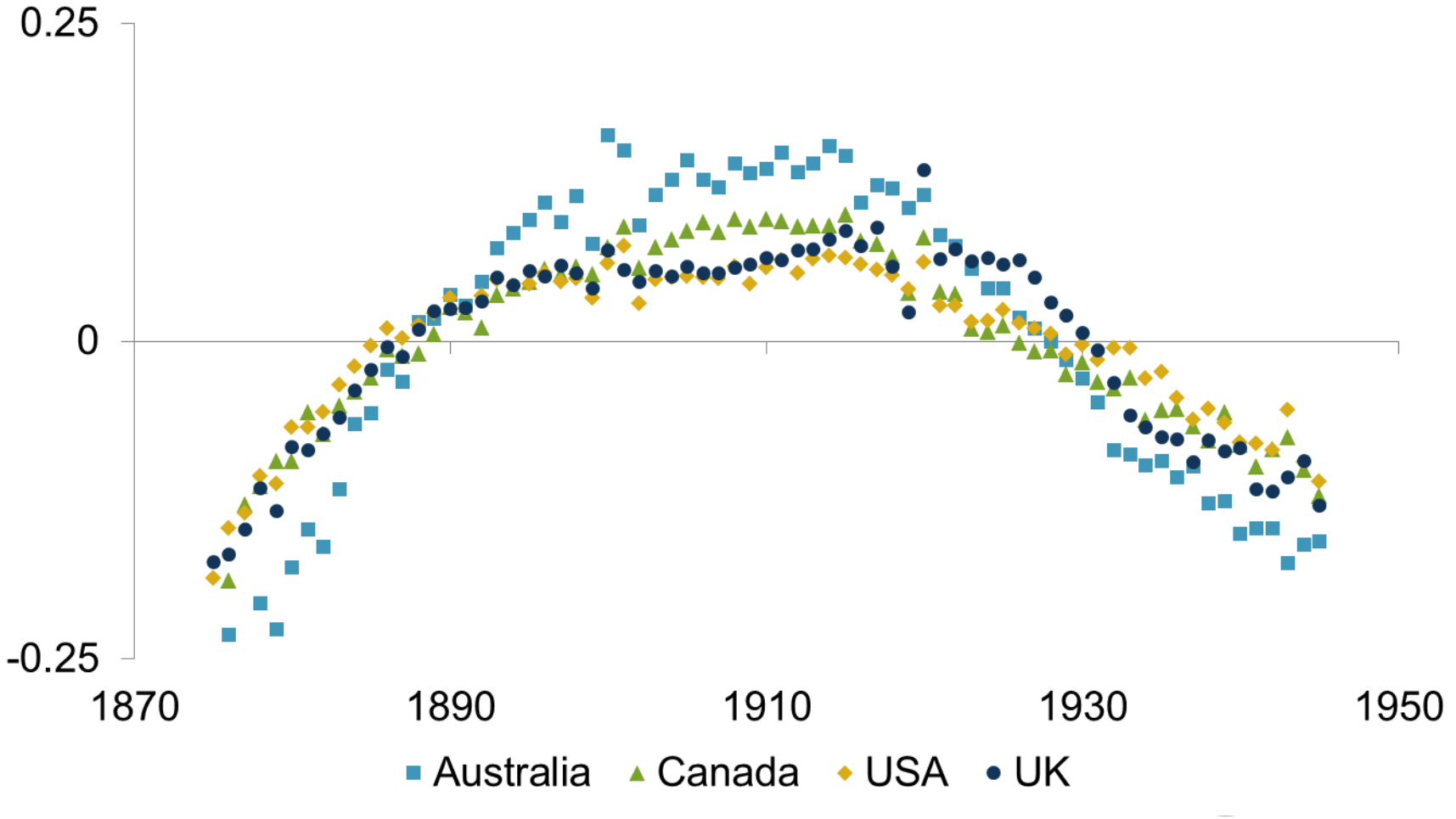
APC cohort parameters – males



APC cohort parameters – females



IMPLICATIONS FOR PROJECTIONS

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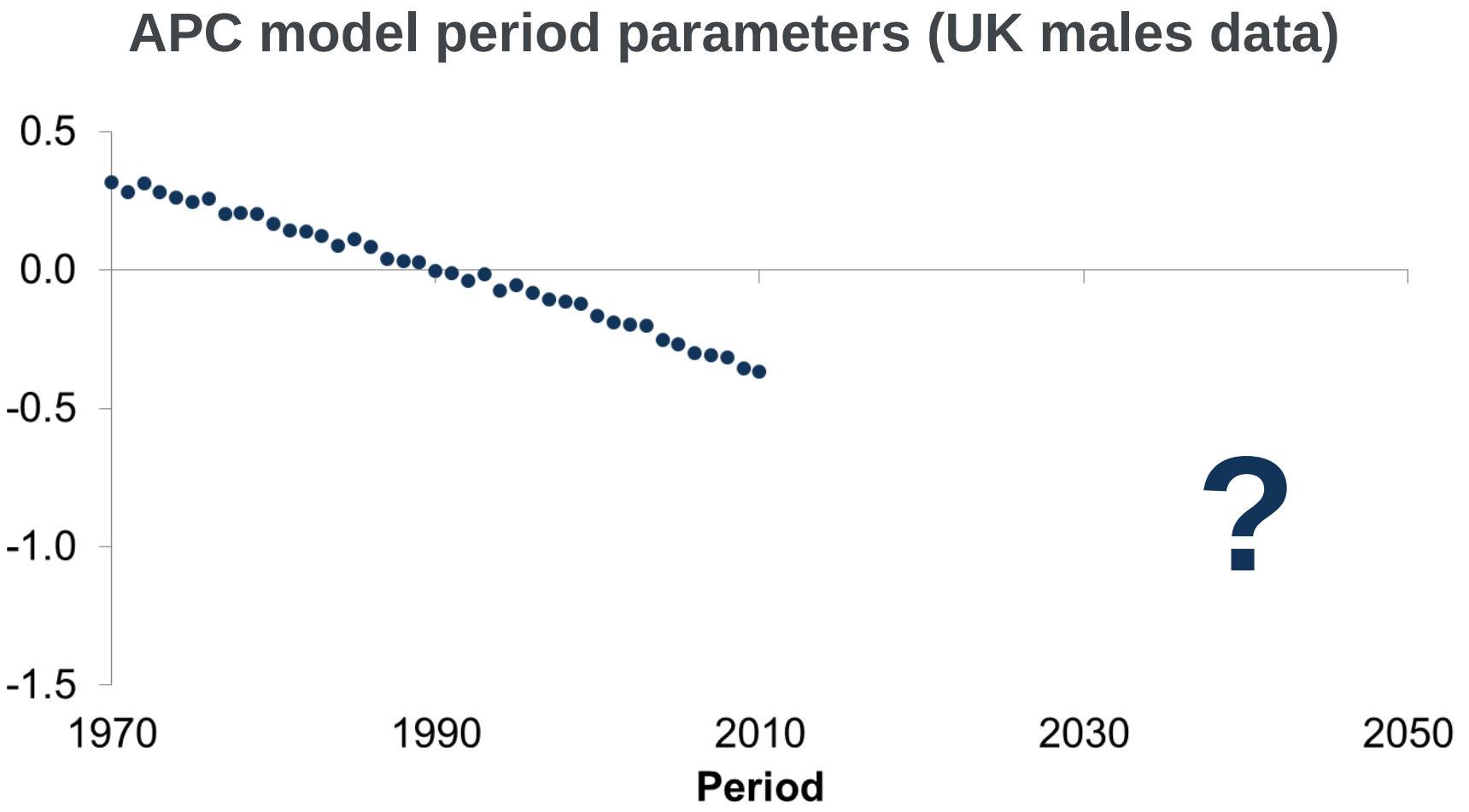
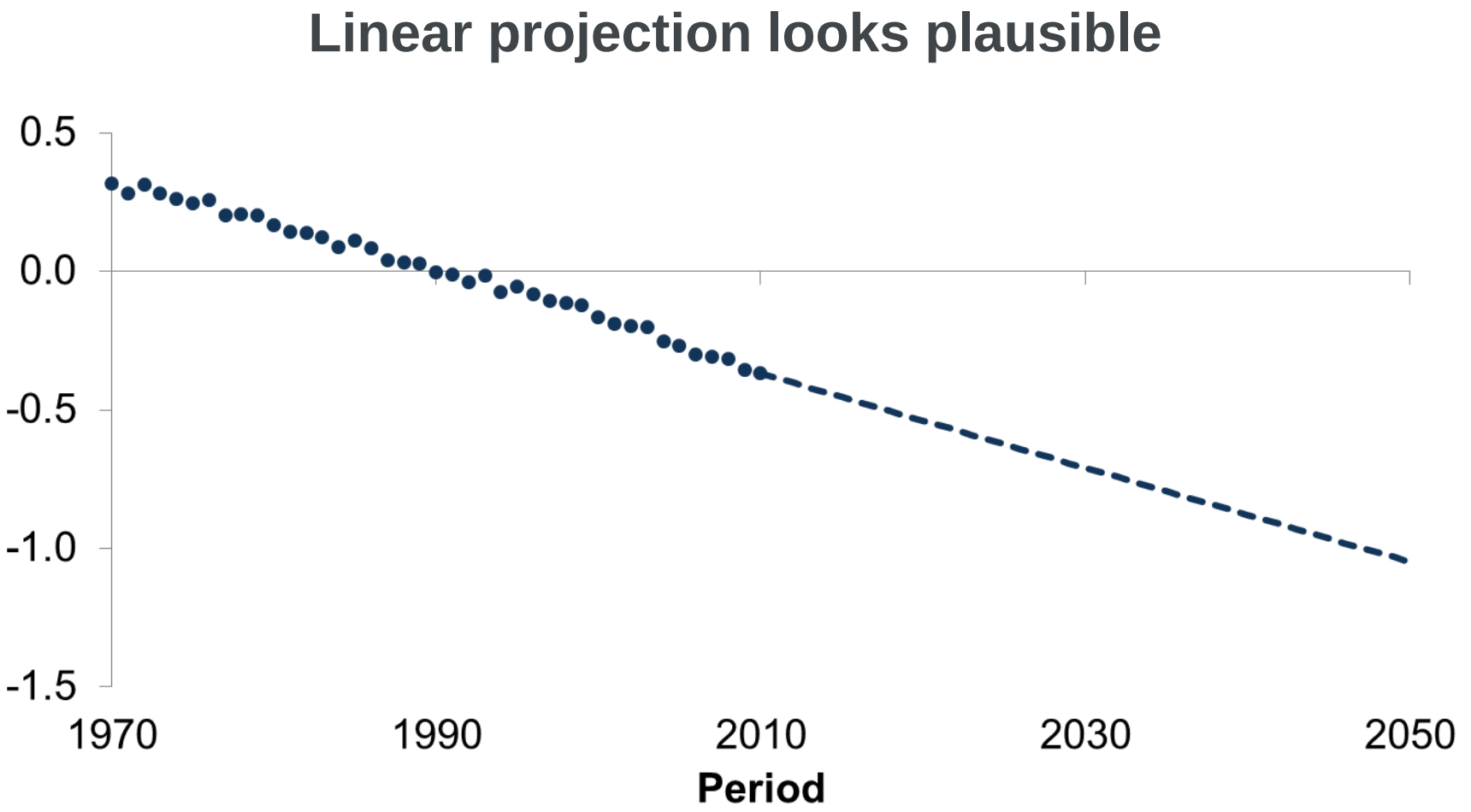
Projecting period parameters

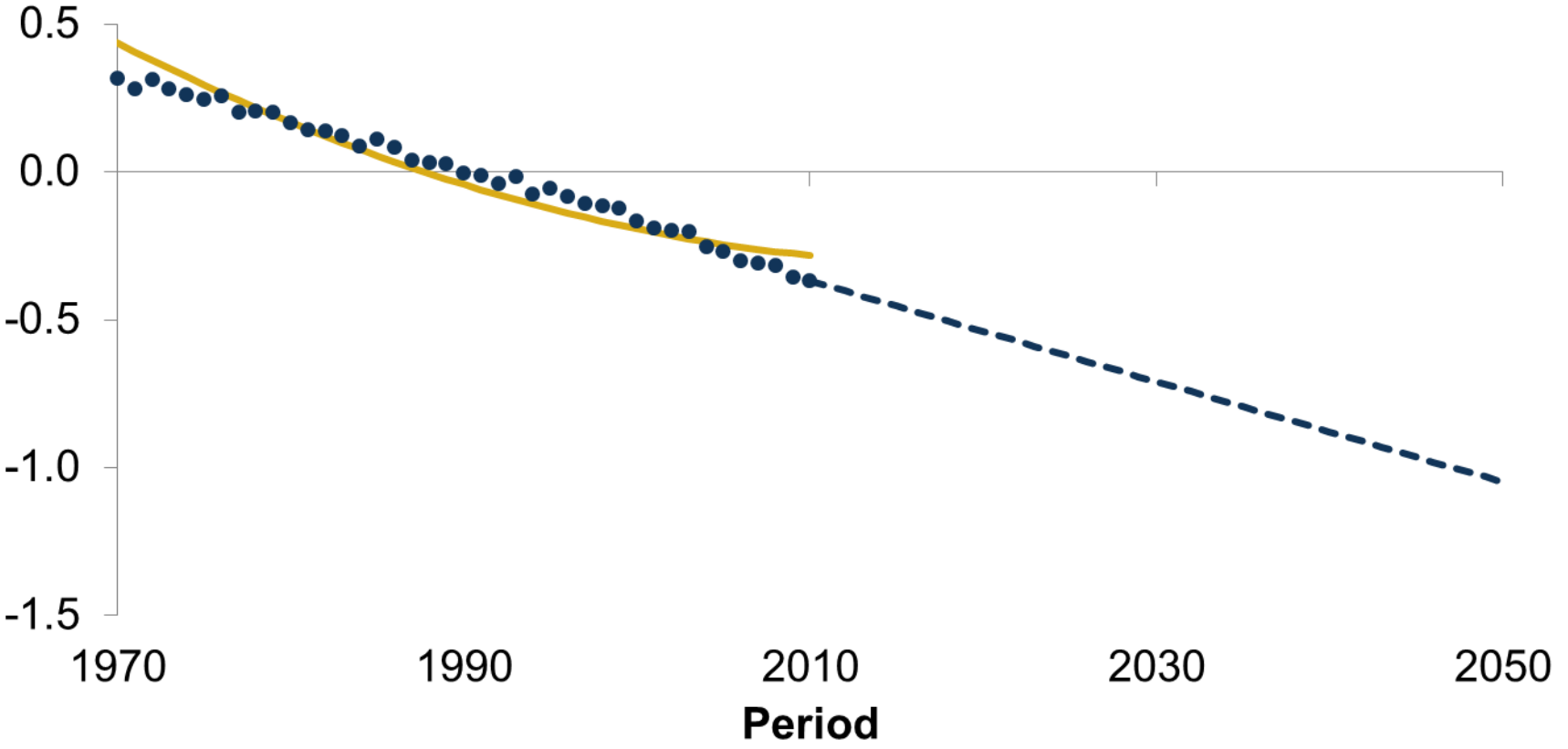


Projecting period parameters



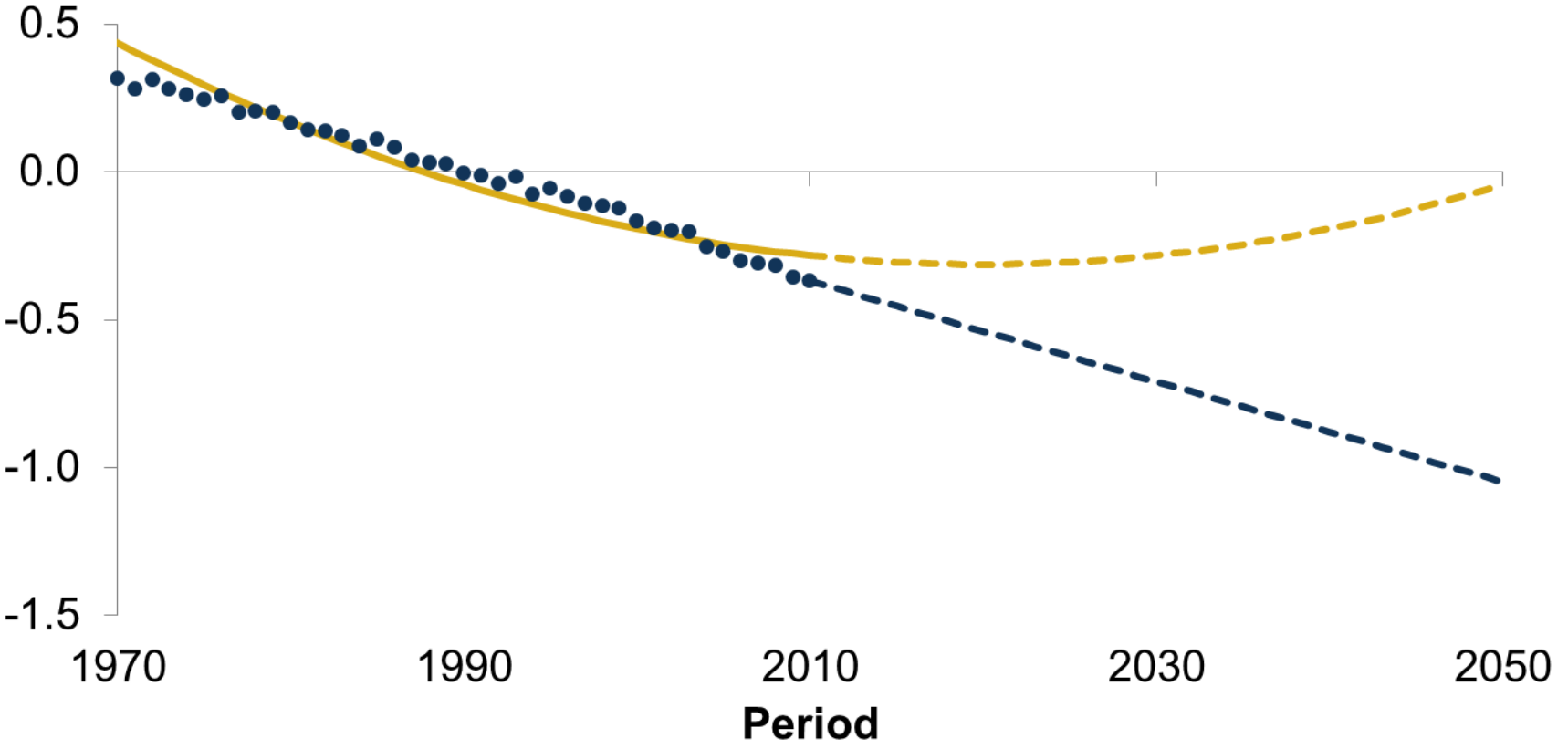
Projecting period parameters

But fitting to synthetic data gives a quadratic



Projecting period parameters

But fitting to synthetic data gives a quadratic



Shape of period parameters

- APC period parameters, when fitted to synthetic data, are

$$\kappa_t = -C(t-\bar{t}) + \frac{1}{2}D(t-\bar{t})^2 - \frac{1}{24}D\left({N_t}^2 - 1\right)$$

- i.e. the shape is influenced by the slope of mortality improvements by age, D (=0.06%)

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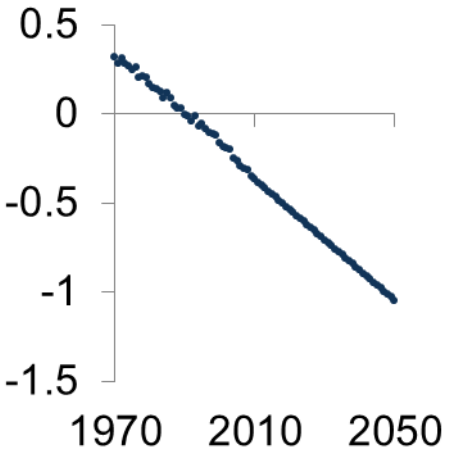
Projecting period parameters

Fit to real data

roughly-linear trend



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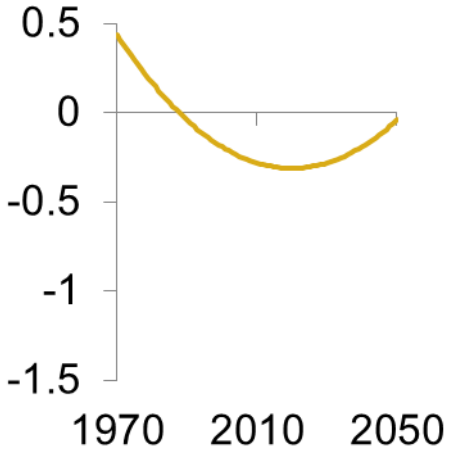
Fit to synthetic data

spurious quadratic element



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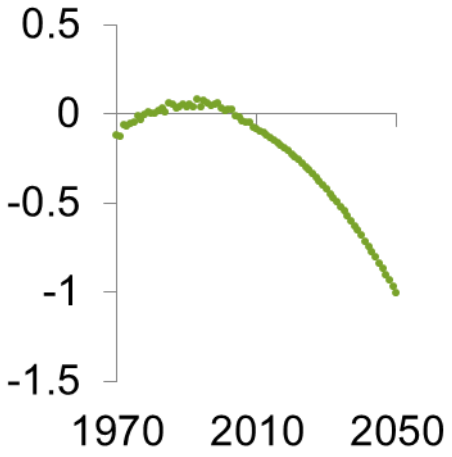
Difference

?



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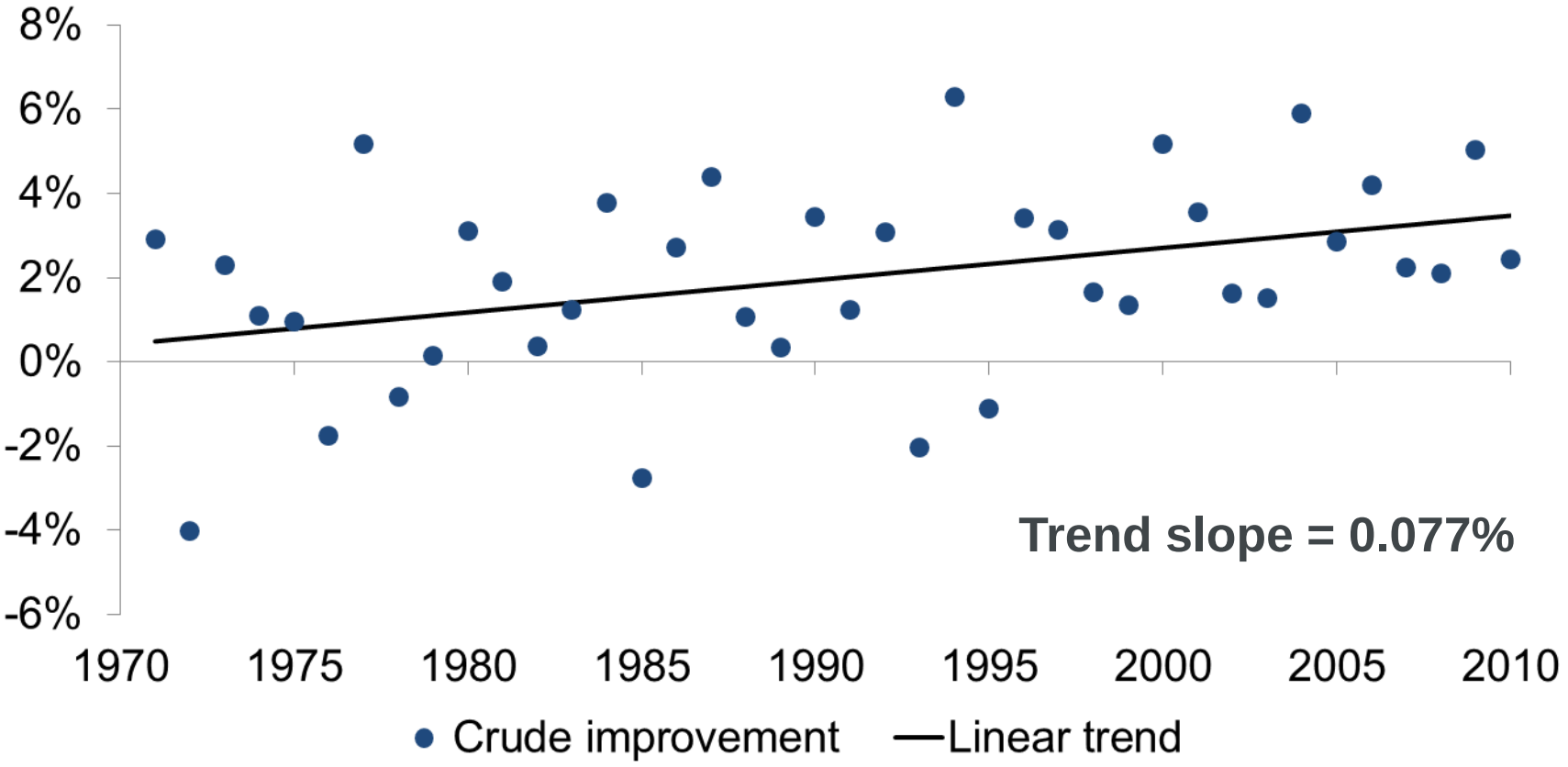
Historical mortality improvements

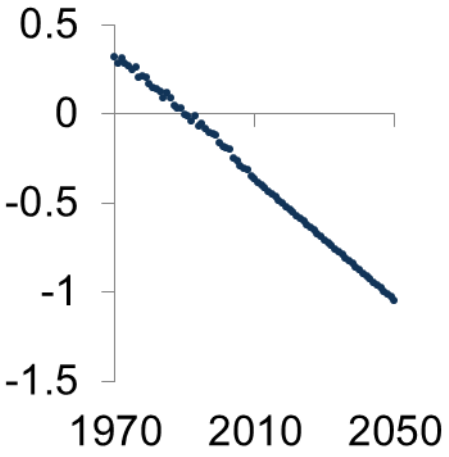
Derived from standardised mortality ratios, ages 60-100



Projecting period parameters

Fit to real data

roughly-linear trend



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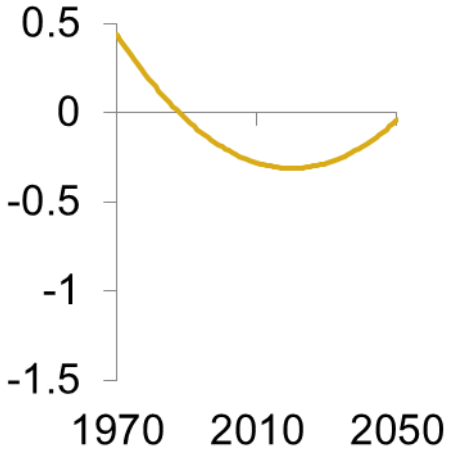
Fit to synthetic data

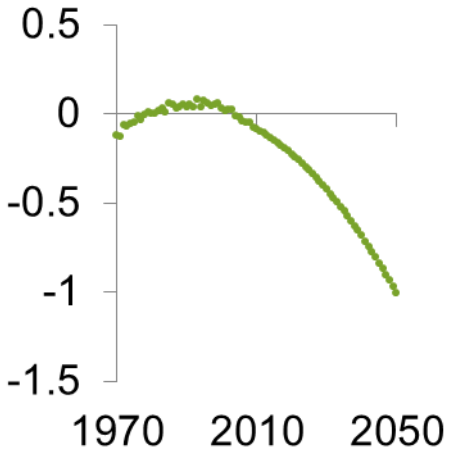
spurious quadratic element



+

Difference

increasing improvements



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Projecting period parameters

- Period parameters are a combination of:
  1. a spurious quadratic term (curving up)
     - This allows the APC model to cope with mortality improvements varying by age. The second derivative is +0.06% (=D, minus the slope of mortality improvements by age)
  2. increasing mortality improvements (curving down)
     - The second derivative is −0.077% (minus the rate of increase of mortality improvements over time)
- The second derivatives nearly cancel out, so the period parameters appear to be linear.

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Projecting period parameters

- Period parameters are a combination of:
  1. a spurious quadratic term (curving up)
  2. increasing mortality improvements (curving down)
- Projecting period parameters linearly assumes that quadratic components of (1) and (2) cancel out in future.
- Under this assumption, the period component of mortality improvements increase indefinitely at a rate of 0.06% (D).

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APC model in Richards et al (2014)

- In discussion of Richards et al (2014), Andrew Smith noted that the average projected annuity from the APC model was outside the confidence interval for the CBD model.
- Our analysis suggests why this happens – linear projection of the APC model period terms gives unusually high mortality improvements.

Annuity values at age 70

Model	Average	1-in-200
CBD (G)	11.98	**12.44**
CBD (P)	11.89	**12.36**
APC	**12.61**	13.04

Note: CBD (G) and CBD (P) are Gompertz and p-spline versions of the CBD M5 model.

Source: Extract from Table 5 of Richards et al (2014)

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Prudential Regulation Authority (PRA) Quantitative Indicators (QIs)

- "based on output from the four families of stochastic models that are considered under the PRA's methodology"
- "model families used are […]: Lee Carter, p-spline, Age-period-cohort (APC) and Cairns, Blake and Dowd (CBD)."
- "differences between the 50th percentile calibrations implied by the four models it investigated as a proxy for the impact of event risk"

Source: Woods (2016), "Reflections on the 2015 Solvency II internal model approval process"

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Prudential Regulation Authority (PRA) Quantitative Indicators (QIs)

- The PRA use “differences between the 50th percentile calibrations implied by the four models it investigated as a proxy for the impact of event risk”
- If results from the APC model are unreliable, and the highest of the four models, then the differences between the four models, and hence the QIs, will be unreliable.
- The PRA are not prepared to disclose further details of their method.

MODELS WITH PLAUSIBLE PARAMETERS

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Which models to use?

- No cohort
  - Lee-Carter, Cairns-Blake-Dowd (CBD) M5
- Spurious cohort (seen in an earlier section)
  - APC, Lee-Carter with cohort, CBD M6, Plat

- We'll now consider
  - CBD M7 (Cairns et al, 2009)
  - CBD M9 (Cairns et al, 2014)
  - APCI (CMI, 2016)

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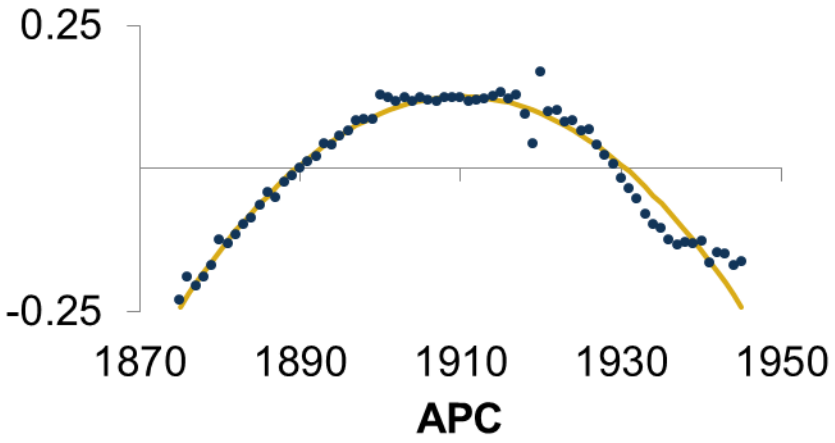
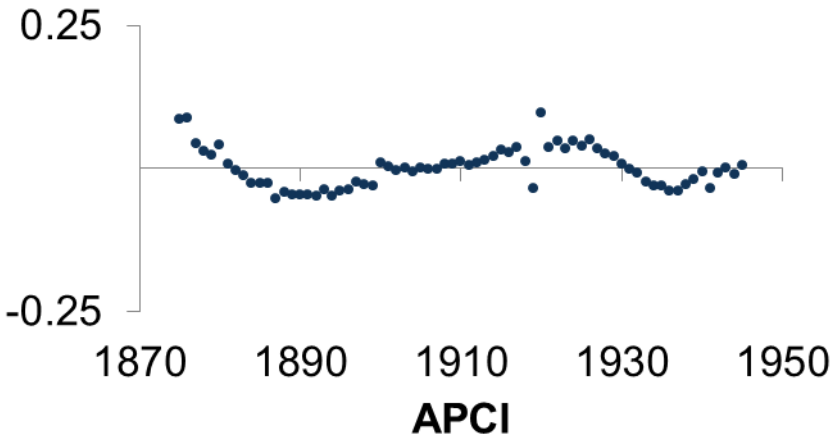
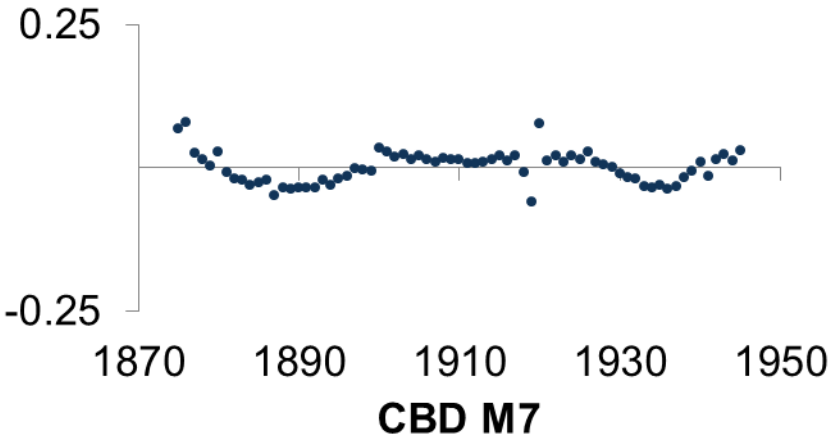
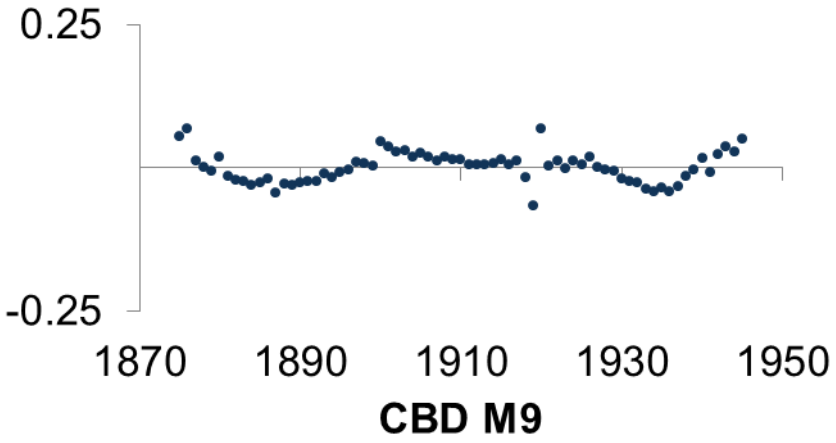
Cohort parameters for APCI, M7 and M9







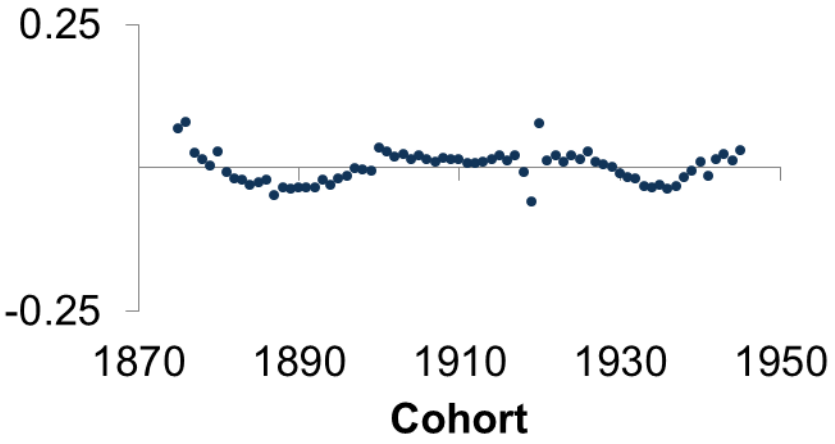


Cohort for M7 fitted to wide age range

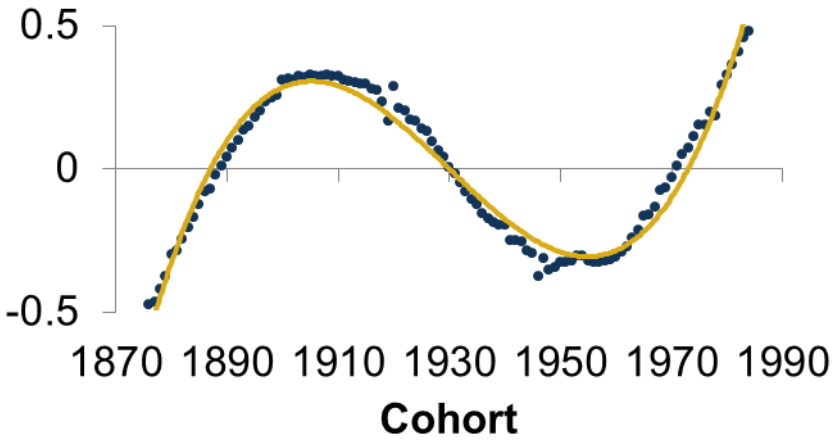
When fitted to ages 60-100, the cohort looks plausible.

When fitted to ages 20-100, it has a spurious cubic shape.

The cohort is being used to model patterns by age.



(different y-axis scale)



Which models to use?

- No cohort – Lee-Carter, Cairns-Blake-Dowd (CBD) M5
- Spurious cohort – APC, Lee-Carter+cohort, CBD M6, Plat
- Struggles with a wide age range – CBD M7

- Plausible over a wide age range (?) – CBD M9, APCI

- Not many left – difficult to assess model risk

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SUMMARY

Summary

- Cohort effects exist; so it's natural to want to model them.
- Fitting to synthetic data shows that cohort parameters need not reflect cohort effects – they can be "spurious".
- Spurious parameters lead to spurious projections.
- Many models have spurious parameters.
- Of the models we considered, only CBD M9 and APCI seem to avoid spurious parameters.
- Model risk is material with only two or three good models.

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Sources

- All data used is from the Human Mortality Database
  - University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany).
  - Available at www.mortality.org or www.humanmortality.de
  - Data downloaded on 31 May 2016.
- Own calculations for UK males unless stated otherwise

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- Woods S (2016) “Reflections on the 2015 Solvency II internal model approval process”, http://www.bankofengland.co.uk/pra/Documents/solvency2/edletter15jan2016.pdf

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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